Math 150, Section 2.4

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Transformations of functions

The purpose of this chapter is to learn how to graph complex functions using the knowledge of graphing simple functions. For example if you know how to graph the function $y = x^2$ you will be able to graph $y = -3(x-4)^2 + 11$. The best way to understand a transformation is to pick a point in the basic graph and see what happens to the point as you apply the transformations one by one.

Vertical shifting

Theorem

If $c > 0$, then the graph of $y = f(x) + c$ is that of $y = f(x)$ shifted c units upward and the graph of $y = f(x) - c$ is that of $y = f(x)$ shifted c units downward.

Try problems 2.4.1a, 2.4.2b.

Horizontal shifting

Theorem

If $c > 0$, then the graph of $y = f(x + c)$ is that of $y = f(x)$ shifted c units to the left and the graph of $y = f(x - c)$ is that of $y = f(x)$ shifted c units to the right.

The fact that adding a constant to x shifts the graph to the left might sound counter intuitive but its not if you think for a while.

Try problems 2.4.1b, 2.4.2a.

Combining horizontal and vertical shifts

We can combine horizontal shift and vertical shifts to get new graphs. For example the graph of $y = (x-1)^3 + 11$ can be obtained from the graph of $y = x^3$ by first shifting it up by 11 units and then to the right 1 units. Here the order does not matter since in either case we will end up with the same graph. This will always be the case when we combine shifts.
Try problem 2.4.7.

Reflecting

Theorem

The graph of $y = -f(x)$ is that of $y = f(x)$ reflected on the x-axis and the graph of $y = f(-x)$ is that of $y = f(x)$ reflected on the y-axis.

The reason for the above theorem is: a point (a, b) is on the graph of $y = f(x)$ if and only if the point $(a, -b)$ is on the graph of $y = -f(x)$ and the points (a, b) and $(a, -b)$ are reflections of one another on the x-axis. You can similary convince yourself the second part of the theorem.
Try problems 2.4.4, 2.4.6a.

Vertical stretching and shrinking

Theorem

If $c > 1$ then the graph of $y = cf(x)$ is that of $y = f(x)$ stretched vertically by a factor of c and if $0 < c < 1$ then the graph of $y = cf(x)$ is that of $y = f(x)$ shrunk vertically by a factor of c.

Try problems 2.4.5, 2.4.8.

Horizontal stretching and shrinking

Theorem

If $c > 1$ then the graph of $y = f(cx)$ is that of $y = f(x)$ shrunk horizontally by a factor of $\frac{1}{c}$ and if $0 < c < 1$ then the graph of $y = cf(x)$ is that of $y = f(x)$ stretched horizontally by a factor of $1/c$.

Try problems 2.4.9, 2.4.10.

Even and odd functions

Theorem

A function f is said to be **even** *if $f(-x) = f(x)$ for all x in the domain of f.*
A function f is said to be **odd** *if $f(-x) = -f(x)$ for all x in the domain of f.*

A given function may be even, odd or neither. To test whether a given function is even or odd algebraically one starts by replacing x with $-x$ in the formula for f, rewrites it and then compares it with f.
Try problems 2.4.61, 2.4.62, 2.4.63, 2.4.68.