Math 150, Section 2.7

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Purpose of this section

The purpose of this section is to study combination of functions. For example if we know the functions $f(x)$ and $g(x)$ how to define the function fg, what can we say about its domain, graph etc.

Algebra of functions

Let f and g be functions with domains A and B. Then the functions $f+g$, $f-g$, fg, and f/g are defined as follows.

$$\begin{aligned}
(f+g)(x) &= f(x)+g(x) \qquad \text{Domain } A \cap B \\
(f-g)(x) &= f(x)-g(x) \qquad \text{Domain } A \cap B \\
(fg)(x) &= f(x)g(x) \qquad \text{Domain } A \cap B \\
(f/g)(x) &= f(x)/g(x) \qquad \text{Domain } \{x \in A \cap B | g(x) \neq 0\}
\end{aligned}$$

Composition of functions

Given two functions f and g, the **composite function** $f \circ g$ (also called the **composition** of f and g) is defined by

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ is all the x in the domain of g such that $g(x)$ is in the domain of f. This is so because if we want to find $f \circ g$ at some point a then first we should be able to find $b = g(a)$ and then we should be able to find $f(g(a)) = f(b)$, hence a should be in the domain of g and $b = g(a)$ should be in the domain of f.

One can similarly define $f \circ g \circ h$ as $(f \circ g \circ h)(x) = f(g(h(x)))$.

You are required to know the following from this section

- Write formula and find the domain of functions created with algebra of functions.
- Write formula for composition of functions given the formula for the individual functions.
- Find the domain of the composition of functions.
- Write a given function as the composition of two or more functions.
- Model functions using composition.

A common mistake in finding the domain of $f \circ g$

When you are trying to find the domain of $f \circ g$ always work with the domains of f and g instead of working with the formula for $f \circ g$. For example if $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$ then $(f \circ g)(x) = x^2$ and the domain of $f \circ g$ is $\{x | x \neq 0\}$ even though the domain of the function with the formula x^2 is all the real numbers. So if you were just looking at x^2 to find the domain you will get an incorrect answer.

Try problem 2.7.33.