

Math 150, Section 2.8

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The purpose of this section

In this section we define one-to-one functions, learn to test for one-to-one function, to find their inverse and to graph them.

Definition of a one-to-one function

Definition

A function with domain A is called a **one-to-one function** if no two elements of A have the same image, that is,

$$f(x_1) \neq f(x_2) \quad \text{whenever} \quad x_1 \neq x_2.$$

In other words f sends different x values to different y values.

Examples of one-to-one functions are a) $f(x) = 3x - 5$, b)

$$f(x) = x^2, 0 \leq x \leq \infty.$$

Examples of functions which are not one to one are a)

$$f(x) = x^2, -\infty \leq x \leq \infty, \text{ b) } f(x) = |x|.$$

Horizontal Line Test

Not all functions are one-to-one and the test which tells us which functions are one-to-one is called the horizontal line test.

Theorem

A function is one-to-one if and only if no horizontal line intersects its graph more than once.

For example the function $f(x) = |x|$ is not one-to-one since the horizontal line $y = 4$ intersects the graph at $x = -4$ as well as $x = 4$.

Definition of the inverse of a function

Definition

Let f be a one-to-one function with domain A and range B . Then its **inverse function** f^{-1} has domain B and range A and is defined by

$$f^{-1}(y) = x \quad \Leftrightarrow \quad f(x) = y$$

for any y in B

Thus if f sends x to y then f^{-1} sends y to x and vice versa.

Inverse function property (cancellation property)

Theorem

Let f be a one-to-one function with domain A and range B . The inverse function f^{-1} satisfies the following cancellation properties.

$$\begin{aligned}f^{-1}(f(x)) &= x && \text{for every } x \text{ in } A \\f(f^{-1}(x)) &= x && \text{for every } x \text{ in } B.\end{aligned}$$

Conversely, any function f^{-1} satisfying these equations is the inverse of f .

In other words if you apply one of f or f^{-1} to x followed by the other, you will get back to x .

Steps to find the inverse of a One-to-One function

- ▶ Write $y = f(x)$.
- ▶ Solve this equation for x in terms of y (if possible).
- ▶ Interchange x and y . The resulting equation is $y = f^{-1}(x)$.

For example if you want to find the inverse of the function $f(x) = \frac{x-1}{x+1}$ then it's as follows:

$$y = \frac{x-1}{x+1} \Rightarrow (x+1)y = (x-1) \Rightarrow xy + y = x - 1$$

$$\Rightarrow xy - x = -y - 1 \Rightarrow x(y-1) = -y-1 \Rightarrow x = \frac{-y-1}{y-1}$$

Thus the inverse of f is $f^{-1}(x) = \frac{-x-1}{x-1}$.

Graph of f^{-1}

Theorem

The graph of f^{-1} is obtained by reflecting the graph of f in the line $y = x$.

Try problems 2, 4, 5, 8, 9, 12, 15, 17, 19, 23, 25, 26, 30, 36, 38, 41, 46, 50, 51, 54, 55, 57.