

Math 150, Section 3.1

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The purpose of this section

In this function we define a polynomial function of degree n and study its properties.

Definition of a polynomial

Definition

A **polynomial of degree** n is a function of the form

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where n is a nonnegative integer and $a_n \neq 0$.

The numbers a_0, a_1, \dots, a_n are called the **coefficients** of the polynomial. The number a_0 is the **constant coefficient** or **constant term**. The number a_n , the coefficient of the highest power, is the **leading coefficient**, and the term $a_n x^n$ is the **leading term**.

Monomials

A monomial is a function of the form $f(x) = x^n$ where n is a non negative integer. Thus a monomial is a particular kind of polynomial.

Try graphing the monomials and use transformations on them to get graphs of some other polynomials. For example graph $f(x) = -(x - 3)^3 + 4$ by first graphing $f(x) = x^3$ and then applying transformations to it.

End Behavior and the Leading Term

We use the notation $x \rightarrow \infty$ to mean "x becomes large in the positive direction". Similarly we use the notation $x \rightarrow -\infty$ to mean "x becomes large in the negative direction".

Let $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$, then

- ▶ If $a_n > 0$ and n is odd then $P(x) \rightarrow \infty$ and $x \rightarrow \infty$.
- ▶ If $a_n > 0$ and n is odd then $P(x) \rightarrow -\infty$ and $x \rightarrow -\infty$.
- ▶ If $a_n > 0$ and n is even then $P(x) \rightarrow \infty$ and $x \rightarrow \infty$.
- ▶ If $a_n > 0$ and n is even then $P(x) \rightarrow \infty$ and $x \rightarrow -\infty$.
- ▶ If $a_n < 0$ and n is odd then $P(x) \rightarrow -\infty$ and $x \rightarrow \infty$.
- ▶ If $a_n < 0$ and n is odd then $P(x) \rightarrow \infty$ and $x \rightarrow -\infty$.
- ▶ If $a_n < 0$ and n is even then $P(x) \rightarrow -\infty$ and $x \rightarrow \infty$.
- ▶ If $a_n < 0$ and n is even then $P(x) \rightarrow -\infty$ and $x \rightarrow -\infty$.

To find the end behavior of a polynomial

Suppose we want to find the end behavior of $p(x) = -5x^7 + 11x^3 - 8x^2 + 1$. Then we can ignore all lower terms and look at only the leading term $-5x^7$. Now since 7 is an odd number, x^7 will be negative when x is negative and hence $-5x^7$ will be positive when x is negative. Thus we can conclude that $p(x) \rightarrow \infty$ when $x \rightarrow -\infty$. Similarly you can show that $p(x) \rightarrow -\infty$ when $x \rightarrow \infty$.

Using Zeros to Graph Polynomials

If P is a polynomial and c is a real number, then the following are equivalent.

- ▶ c is a zero of P .
- ▶ $x = c$ is a solution of the equation $P(x) = 0$.
- ▶ $x - c$ is a factor of $P(x)$.
- ▶ $x = c$ is an x -intercept of the graph of P .

Intermediate Value Theorem for polynomials

If P is a polynomial function and $P(a)$ and $P(b)$ have opposite signs, then there exists at least one value c between a and b for which $P(c) = 0$.

Steps for Graphing a Polynomial

- ▶ Factor the polynomial to find all its real zeros; these are the x -intercepts of the graph.
- ▶ Make a table of values for the polynomial. Include test points to determine whether the graph of the polynomial lies above or below the x -axis on the intervals determined by the zeros. Include the y -intercept in the table.
- ▶ Determine the end behavior of the polynomial.
- ▶ Plot the intercepts and other points you found in the table. Sketch a smooth curve that passes through these points and exhibits the required end behavior.

Shape of the Graph Near a Zero of Multiplicity m

Please read the colored box in page 259 of the book.

Local Extrema of Polynomials

If $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ is a polynomial of degree n , then the graph of P has at most $n - 1$ local extrema.

You can find the local extrema of a given polynomial by first graphing it.

Try problems 1, 5, 8, 16, 19, 27, 37, 38, 46, 49.