Math 150, Section 3.5

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Purpose of this section

In this section we study the connection between polynomials, its zeros and its factorization. At the end of this section you should be able to find zeros of a given polynomial and construct polynomials with specified zeros.

Fundamental Theorem of Algebra

Theorem

Every polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \qquad (n \geq 1, a_n \neq 0)$$

with complex coefficients has atleast one complex zero.

Since real numbers are also complex numbers, the theorem applies to polynomials with real coefficients as well.

Complete Factorization Theorem

Theorem

If $P(x)$ is a polynomial of degree $n \geq 1$, then there exist complex numbers $a, c_1, c_2, \cdots, c_n$ (with $a_n \neq 0$) such that

$$P(x) = a(x - c_1)(x - c_2) \cdots (x - c_n).$$

The $c_1, c_2, \cdots, c_n$ are obtained by finding the zeros of $P(x)$.

Conjugate Zeros Theorem

Theorem

If the polynomial P has real coefficients, and if the complex number $a + bi$ is a zero of P, then its complex conjugate $a - bi$ is also a zero of P.

Thus the complex zeros occur in pairs. So if you are asked to construct a polynomial with real coefficients with zeros 1 and $2 + 3i$ then its implied that $2 - 3i$ will also be a zero of that polynomial.

Linear and Quadratic Factors Theorem

Theorem

Every polynomial with real coefficients can be factored into a product of linear and irreducible quadratic factors with real coefficients.

This theorem does not have any explicit application to problems.

Problems

Try problems 1, 6, 12, 18, 37, 59.