Math 150, Section 3.6

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Purpose of this section

In this section we study rational functions, define certain properties of rational functions and learn how to find them and graph them. In the end you should be able to do the following

- Tell whether a given function is rational or not.
- Find x and y intercepts.
- Find vertical asymptotes and the behavior of the function near them.
- Find horizontal asymptotes.
- Find slant asymptotes.
- Graph the function showing all of the above.

Rational Function

A function $f(x)$ of the form $f(x) = \frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials is called a rational function. Thus the functions $3x, \dfrac{3}{x}, \dfrac{x^2+1}{x-1}$ are all rational functions and $\dfrac{x^2+1}{\sqrt{x-1}}$ is not.

Intercepts

The intercepts are the points on the corresponding axis where the graph intersects. The x-intercept of a rational function is found by finding the zeros of the numerator(because every point on the x-axis has y value 0) and y-intercept is found by plugging in $x = 0$ to the function(similar reasoning).

Vertical Asymptote

The line $x = a$ is said to be a vertical asymptote if $y \to \pm\infty$ as $x \to a$ from the left or from the right. Thus the function $f(x) = \frac{x^2}{2x+4}$ has the vertical asymptote $x = -2$.

A given rational function can have many vertical asymptotes. The vertical asymptotes are found by setting the denominator equal to zero and solving for x.

Left and Right limits and behavior near VA

We use the notation $x \to a^-$ to denote "x approaches a from left" and $x \to a^+$ to denote "x approaches a from right".

To find the behavior of a rational function near the vertical asymptote $x = a$, you will have to make the sign chart of the function near a.

Horizontal Asymptote

The line $y = b$ is said to be a horizontal asymptote if $y \to b$ as $x \to \infty$ or $x \to -\infty$. Thus the function $f(x) = \frac{x^2}{2x^2+4}$ has the horizontal asymptote $y = 1/2$.
A given rational function may or may not have a horizontal asymptote. If it does then it will be unique. The only time a rational function will have a horizontal asymptote is when the degree of the numerator and denominator are equal and in this case the horizontal asymptote is found by dividing the leading coefficients.

Slant Asymptote

If a rational function $r(x)$ has the form $r(x) = ax + b + \frac{R(x)}{Q(x)}$ where $a \neq 0$ and degree of R is less than degree of Q, then we say that $y = ax + b$ is a slant asymptote of $r(x)$.

Thus a rational function will have a slant asymptote if the degree of the numerator is exactly one more than the degree of the denominator and we can find the asymptote using long division.

Putting it all together

Given a rational function you should be able to find all the properties we described above and graph the function.