Math 150, Section 5.1

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Purpose of This Section

In this section we learn the following

- Definition of degrees and radians and relation between them
- Coterminal angles
- Finding the length of a circular arc and area of a circular sector
- Definition of linear speed and angular speed and the relation between them

Definition of angle

An angle consists of two rays called **initial side** and **terminal side** and the angle is thought of obtained by rotating the initial side to the terminal side. If the rotation is counter clockwise then the angle is positive and it is negative otherwise. The units used to measure angles are **radians** and **degrees**.

Degree and Radian

One full rotation is defined to be equivalent of 360^o. Thus 1 **degree** is defined as $\frac{1}{360}$ of a full rotation.

One **radian** is defined as the angle subtended by an arc of length one of a circle of radius one at its center. Thus a full rotation is equivalent to 2π radians since the perimeter of a full circle of radius one is 2π.

Since a full rotation in radians is 2π and in degrees is 360^o we have the formula π radians $= 180^o$. Thus we get the conversion formulas

$$1 \text{ radian} = \left(\frac{180}{\pi}\right)^o \qquad 1^0 = \frac{\pi}{180} \text{ radians}$$

Coterminal Angles

We say two angles are **coterminal** if the the terminal sides coincide whenever the initial sides coincide. Thus the difference of the angles will be a multiple of 360 if in degrees and 2π if in radians.

Length of a Circular Arc and Area of a Circular Sector

The length of a circular arc which subtends an angle θ radians at the center in a circle of radius r is given by the formula $s = r\theta$.

The area of a circular sector which subtends an angle θ at the center in a circle of radius r is given by the formula $A = \frac{1}{2}r^2\theta$.

Linear Speed and Angular Speed

Suppose a point moves along a circle of radius r. Let s be the arc length that the object covers in time t and let θ be the angle that the object traverses in time t. We define the **linear speed** of the object to be $v = \frac{s}{t}$ and the **angular speed** to be $w = \frac{\theta}{t}$.