

## Math 150, Section 6.2

Lec: Naushad Pasha Puliyambalath

November 8, 2009

## Purpose of This Section

In this section we learn the following

- ▶ Definition of trigonometric functions and their domains
- ▶ The signs of each trigonometric function in each quadrant
- ▶ Even and odd properties of the trigonometric functions and some trigonometric identities
- ▶ Finding the exact value of the trigonometric functions using reference number

## Trigonometric Functions of Real Number

In section 5.2 we defined the trigonometric ratios for any  $\theta$  between 0 and  $\pi/2$ . Now we extend this to all real numbers and called the corresponding functions as trigonometric functions.

Given a real number  $t$ , let  $P(x, y)$  be the terminal point determined by  $t$ . We define

$$\sin t = y \quad \cos t = x \quad \tan t = \frac{y}{x}$$

$$\csc t = \frac{1}{y} \quad \sec t = \frac{1}{x} \quad \cot t = \frac{x}{y}$$

When  $t$  is between 0 and  $\pi/2$ , the trigonometric function value coincides with the corresponding trigonometric ratio.

## Domains of Trigonometric Functions

- ▶ The domain of sin and cos are all the real numbers
- ▶ The domain of tan and sec are all the real numbers except  $\frac{\pi}{2} + n\pi$  where  $n$  is an integer
- ▶ The domain of cot and csc are all the real numbers except  $n\pi$  where  $n$  is an integer

## Signs of The Trigonometric Functions

- ▶ **A**ll the trigonometric functions are positive in the first quadrant.
- ▶ Only **s**ine and **c**osine are positive in the second quadrant
- ▶ Only **t**angent and **c**otangent are positive in the third quadrant
- ▶ Only **c**osine and **s**ecant are positive in the fourth quadrant

You can use the phrase "All Students Take Calculus" to remember the above conditions.

## Even and Odd Properties

Cosine and secant are the only even trigonometric functions.

Since the point  $(\cos t, \sin t)$  is a point on the unit circle (terminal point determined by  $t$ ), it satisfies the equation of the unit circle and hence we have the important identity

$$\sin^2 t + \cos^2 t = 1$$

Dividing this identity both sides with  $\cos t$  and rewriting gives

$$1 + \tan^2 t = \sec^2 t$$

Dividing with  $\sin t$  gives

$$1 + \cot^2 t = \csc^2 t$$