Math 150, Section 7.4

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Purpose of This Section

In this section we learn the following

- Definition of the inverse sine, cosine and tangent functions
- Composing trigonometric functions and their inverses

Inverse Sine Function

The **inverse sine function** is the function $\sin^{-1}$ with domain $[-1, 1]$ and range $[-\frac{\pi}{2}, \frac{\pi}{2}]$ defined by

$$\sin^{-1} x = y \quad \Leftrightarrow \quad \sin y = x$$

The inversse sine function is also called **arcsine**, denoted by arcsin.

From the definition of inverse function we have

$$\sin(\sin^{-1} x) = x \qquad \text{for } -1 \leq x \leq 1$$

$$\sin^{-1}(\sin x) = x \qquad \text{for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

Inverse Cosine Function

The **inverse cosine function** is the function $\cos^{-1}$ with domain $[-1, 1]$ and range $[0, \pi]$ defined by

$$\cos^{-1} x = y \quad \Leftrightarrow \quad \cos y = x$$

The inversse cosine function is also called **arccosine**, denoted by arccos.

From the definition of inverse function we have

$$\cos(\cos^{-1} x) = x \qquad \text{for } -1 \leq x \leq 1$$

$$\cos^{-1}(\cos x) = x \qquad \text{for } 0 \leq x \leq \pi$$

Inverse Tangent Function

The **inverse tangent function** is the function $\tan^{-1}$ with domain $(-\infty, \infty)$ and range $(-\frac{\pi}{2}, \frac{\pi}{2})$ defined by

$$\tan^{-1} x = y \quad \Leftrightarrow \quad \tan y = x$$

The inversse tangent function is also called **arctangent**, denoted by arctan.

From the definition of inverse function we have

$$\tan(\tan^{-1} x) = x \qquad \text{for } -\infty \leq x \leq \infty$$

$$\tan^{-1}(\tan x) = x \qquad \text{for } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

Composing Trigonometric Functions and Their Inverses

It is possible to rewrite certain composition of trigonometric functions and their inverses without using any trigonometric functions. For example one can rewrite $\sin(2\cos^{-1} x)$ as $2x\sqrt{1 - x^2}$. These types of problems are important.