Math 131
Autumn 2009
Midterm 3
Form A

Name: ______________________

Name.nn: ______________________

Lecturer: ______________________

Rec. Instructor: ______________________

Rec. Time: ______________________

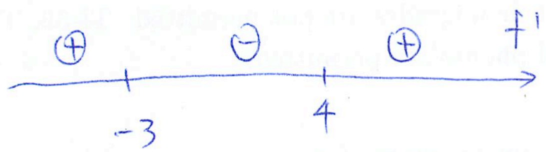
Instructions:

- You have **48 minutes** to complete this exam. It consists of 7 problems on 7 pages including this cover sheet. It is worth a total of 100 points. The value of each question is listed below.
- You may not use any books or notes during this exam.
- Calculators are permitted EXCEPT those calculators that have symbolic algebra or calculus capabilities. In particular, the following calculators and their upgrades are not permitted: TI-89, TI-92, and HP-49. In addition, neither PDAs, laptops nor cell phones are permitted.
- Make sure to read each question carefully.
- Please **write clearly** and make sure to **justify your answers**. Correct answers with no supporting work may receive no credit.
- Please write your answers on the indicated lines.
- A random sample of graded exams will be xeroxed before being returned.

Problem	Point Value	Score
1	10	
2	10	
3	26	
4	18	
5	16	
6	10	
7	10	
Total	100	

(1.) Let $f(x) = 2x^3 - 3x^2 - 72x$. Use derivatives and a sign graph to determine the interval(s) on which $f(x)$ is increasing and on which $f(x)$ is decreasing. (If there are none, please say so).

$$f'(x) = 6x^2 - 6x - 72$$
$$= 6(x^2 - x - 12)$$
$$= 6(x-4)(x+3)$$
$$x = 4 \quad x = -3.$$

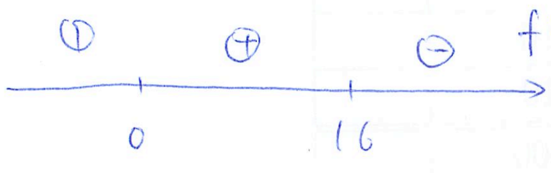


Answer (1.): increasing: $(-\infty, -3) \cup (4, \infty)$

decreasing: $(-3, 4)$

(2.) Let $f(x) = -x^6 + 24x^5 - 24x + 17$. Use derivatives and a sign graph to determine the interval(s) on which $f(x)$ is concave up and on which $f(x)$ is concave down. (If there are none, please say so).

$$f'(x) = -6x^5 + 120x^4 - 24$$
$$f''(x) = -30x^4 + 480x^3 = -30x^3(x-16)$$



Answer (2.): concave up: $(0, 16)$

concave down: $(-\infty, 0) \cup (16, \infty)$

(3.) Let $f(x) = 10x^6 - 13x^5 + 13$.

(a.) Find y-intercept of $f(x)$.

Answer (3a.): y-intercept: 13

(b.) Use derivatives and a sign graph to determine the interval(s) on which $f(x)$ is increasing and on which $f(x)$ is decreasing AND indicate where $f(x)$ has local maximum and local minimum points. (If there are none, please say so).

$$f'(x) = 60x^5 - 65x^4$$
$$= 60x^4\left(x - \frac{65}{60}\right)$$
$$= 60x^4\left(x - \frac{13}{12}\right)$$

Answer (3b.): increasing: $\left(\frac{13}{12}, \infty\right)$

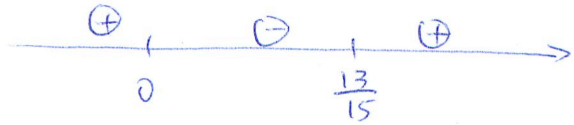
decreasing: $(-\infty, 0) \cup \left(0, \frac{13}{12}\right)$

local max. points(s) at $x =$ None

local min. point(s) at $x =$ $\frac{13}{12}$

(c.) Use derivatives and a sign graph to determine the interval(s) on which $f(x)$ is concave up and on which $f(x)$ is concave down AND indicate where $f(x)$ has inflection point(s). (If there are none, please say so).

$$f''(x) = 300x^4 - 260x^3$$
$$= 300x^3\left(x - \frac{260}{300}\right)$$
$$= 300x^3\left(x - \frac{13}{15}\right)$$



Answer (3c.): concave up: $(-\infty, 0) \cup \left(\frac{13}{15}, +\infty\right)$

concave down: $\left(0, \frac{13}{15}\right)$

inflection point(s) at $x =$ $0, \frac{13}{15}$

(4.) Let $f(x) = \frac{5x^2 + 10}{x^2 - 36}$

(a.) Find it the x-intercept(s) and y-intercept of $f(x)$.

Answer (4a.): x-intercept(s): None

y-intercept: $-\frac{10}{36}$

(b.) Find all horizontal asymptotes of $f(x)$. (If there are none, please say so).

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{5x^2}{x^2} = 5$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{5x^2}{x^2} = -5$$

Answer (4b.): horizontal asymptote(s): $y = 5$.

(c.) Find all vertical asymptotes of $f(x)$. (If there are none, please say so).

$$x^2 - 36 = 0$$

$$x^2 = 36$$

$$x = \pm 6$$

Answer (4c.): vertical asymptote(s): $x=6$ $x=-6$

(5.) Let $f(x) = x^3 - 9x^2 + 15x + 13$.

(a.) Use the Second Derivative Test to find where the local maximum(s) and the local minimum(s) of $f(x)$ occur.

$f'(x) = 3x^2 - 18x + 15$

$= 3(x^2 - 6x + 5)$

$= 3(x-1)(x-5)$

$x = 5 \quad x = 1$

$f''(x) = 6x - 18$

$f''(1) = -12$

$f''(5) = 12$

Answer (5a.): local max(s). at $x =$ 1

local min(s). at $x =$ 5

(b.) Find where the absolute maximum and absolute minimum for $f(x)$ occur over the interval [4, 7].

Evaluate f at $x = 4, 5, 7$.

$f(4) = -7$

$f(5) = -12$

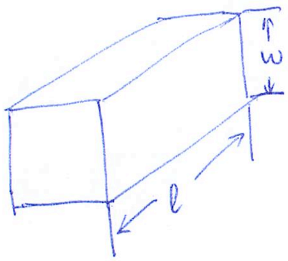
$f(7) = 20$

Answer (5b.): absolute max(s). at $x =$ 7

absolute min(s). at $x =$ 5

(6.) A parcel delivery service will deliver a package only if the length plus girth (distance around) does not exceed 84 inches. Find the dimensions of a rectangular box with square ends that satisfies the delivery service's restrictions and has maximal volume.

To make the box as large as possible.



$l + 4w = 84$.

By default, $l \geq 0$

$w \geq 0$.

so $0 \leq w \leq 21$.

$$V = l \cdot w^2$$
$$= (84 - 4w)w^2$$
$$= 84w^2 - 4w^3$$

$$\frac{dv}{dw} = 168w - 12w^2$$
$$= -12w(w-14)$$

$w = 0 \quad w = 14$

$f(0) = 0$

$f(14) = 5488$

$f(21) = 0$

so $w = 14$, $l = 28$

Answer (6.): dimensions: $w = 14$, $l = 28$

(7.) Use the given information to sketch a graph of $f(x)$:

Domain: All real except $x = -3$ and $x = 2$.

$f(-4) = -2, f(-1) = 0, f(3) = 2$.

$f'(x) < 0$ on $(-\infty, -3)$ and $(2, \infty)$, $f'(x) > 0$ on $(-3, 2)$.

$f''(x) < 0$ on $(-\infty, -3)$ and $(-3, -1)$, $f''(x) > 0$ on $(-1, 2)$ and $(2, \infty)$.

Vertical asymptotes: $x = -3$ and $x = 2$, Horizontal asymptotes: $y = 0$.