1) $\sqrt{4.25}$

Solution. Set $f(x) = \sqrt{x}$.

$$f'(x) = (\sqrt{x})'$$
$$= (x^{\frac{1}{2}})'$$
$$= \frac{1}{2} x^{-\frac{1}{2}}$$

$$f'(2) = \frac{1}{2}(4)^{-\frac{1}{2}}$$
$$= \frac{1}{4}$$

$$\Delta y \approx dy = f'dx = \frac{1}{4} \cdot (4.25-4) = \frac{1}{4} \cdot 0.25 = 0.0625$$

i.e. $f(4.25) - f(4) \approx 0.0625$

so $\sqrt{4.25} = f(4.25) \approx f(4) + 0.0625 = \boxed{2.06250}$

2) $\int (5+x^3)(4-x^2)\,dx$

Solution. $= \int (20 - 5x^2 + 4x^3 - x^5)\,dx$

$$= 20x - 5\frac{x^3}{3} + 4\frac{x^4}{4} - \frac{x^6}{6} + C$$

$$= \boxed{20x - \frac{5}{3}x^3 + x^4 - \frac{x^6}{6} + C}$$

3) $\int (2\sqrt[3]{x^4} - x^{-3})\,dx$

Solution. $= \int (2x^{\frac{4}{3}} - x^{-3})\,dx$

$$= 2\frac{x^{\frac{7}{3}}}{\frac{7}{3}} - \frac{x^{-2}}{-2} + C$$

$$= \boxed{\frac{6}{7}x^{\frac{7}{3}} + \frac{1}{2}x^{-2} + C}$$

4) $\int (2+2x)\, e^{(4x+2x^2)}\, dx$

Solution. substitution $u = 4x + 2x^2$.

$$du = (4+4x)\, dx$$

$$dx = \frac{du}{4+4x}$$

So $\int (2+2x)\, e^{(4x+2x^2)}\, dx$

$$= \int (2+2x)\, e^{u}\, \frac{du}{4+4x}$$

$$= \int \frac{1}{2} e^{u}\, du$$

$$= \frac{1}{2} \int e^{u} du$$

$$= \frac{1}{2} e^{u} + C$$

$$= \boxed{\frac{1}{2} e^{(4x+2x^2)} + C}$$

5) $\int_1^e \left[16x - \frac{5}{x}\right] dx$

Solution.

$$= 8x^2 - 5\ln|x| \Big]_1^e$$

$$= (8e^2 - 5\ln e) - (8\cdot 1^2 - 5\ln 1)$$

$$= 8e^2 - 5 - 8$$

$$= \boxed{8e^2 - 13}$$

6) $\int_0^2 \frac{4x+1}{4x^2+2x+2}\,dx$

Solution.

Substitution.

$u = 4x^2+2x+2$, thus $u(0) = 2$, $u(2) = 22$

$du = (8x+2)dx$

$dx = \frac{du}{8x+2}$.

so $\int_0^2 \frac{4x+1}{4x^2+2x+2}\,dx$

$= \int_2^{22} \frac{4x+1}{u} \frac{du}{8x+2}$

$= \int_2^{22} \frac{1}{2}\frac{1}{u}\,du$

$= \frac{1}{2}\ln|u| \Big]_2^{22}$

$= \frac{1}{2}\ln 22 - \frac{1}{2}\ln 2$

$= \boxed{1.199}$

7) $\int_3^7 \frac{x^2-16}{x-4}\,dx$

Solution.

$= \int_3^7 \frac{(x-4)(x+4)}{x-4}\,dx$

$= \int_3^7 x+4\,dx$

$= \frac{1}{2}x^2+4x \Big]_3^7$

$= \left(\frac{1}{2}\cdot 7^2+4\cdot 7\right) - \left(\frac{1}{2}\cdot 3^2+4\cdot 3\right)$

$= \boxed{36}$

8). Solution.

$$f'(x) = \frac{3}{x^5} = 3x^{-5} \qquad f(\tfrac{1}{2}) = 1.$$

$$f(x) = \int 3x^{-5}\,dx$$

$$= 3\frac{x^{-4}}{-4} + C$$

$$= -\frac{3}{4}x^{-4} + C.$$

Since $f(\frac{1}{2}) = 1$, $\quad 1 = -\frac{3}{4}\left(\frac{1}{2}\right)^{-4} + C$

$$1 = -\frac{3}{4}\cdot 16 + C$$

$$1 = -12 + C$$

$$C = 13.$$

Thus, $\boxed{f(x) = -\frac{3}{4}x^{-4} + 13.}$

9). $\frac{dy}{dx} = \frac{1}{2+x}$, $y(0) = 3$.

Solution.

$$y = \int \frac{1}{2+x}\,dx.$$

$$= \ln|2+x| + C.$$

Since $y(0) = 3$, $\quad 3 = \ln 2 + C$

$$C = 3 - \ln 2.$$

Thus $\boxed{y = \ln|2+x| + 3 - \ln 2.}$

10). Solution.

$$\Delta x = \frac{7-(-3)}{5} = 2.$$

$$x_k = -3 + 2k.$$

$$c_k = \frac{x_{k-1}+x_k}{2} = \frac{-3+2(k-1)+(-3+2k)}{2} = -4+2k.$$

$$S_5 = \sum_{k=1}^{5} f(c_k)\,\Delta x$$

$$= f(c_1)\Delta x + f(c_2)\Delta x + f(c_3)\Delta x + f(c_4)\Delta x + f(c_5)\Delta x$$

$$= \left(f(c_1)+f(c_2)+f(c_3)+f(c_4)+f(c_5)\right)\Delta x$$

$$= \left(f(-2)+f(0)+f(2)+f(4)+f(6)\right)\Delta x$$

$$= (0+(-10)+(-12)+(-6)+8)\cdot 2$$

$$= \boxed{-40.}$$

11) $\int_3^5 f(x)\,dx = 7$ $\int_3^5 g(x)\,dx = 1$. find $\int_3^5 [4f(x)-2g(x)]\,dx$.

Solution. $\int_3^5 [4f(x)-2g(x)]\,dx$

$$= 4\int_3^5 f(x)\,dx - 2\int_3^5 g(x)\,dx$$

$$= 4\cdot 7 - 2\cdot 1$$

$$= \boxed{26}$$

12). $\int_5^7 f(x)\,dx = 6$, $\int_2^5 g(x)\,dx = 2$, $\int_2^7 g(x)dx = \frac{5}{3}$, find $\int_7^5 (4g(x)-2f(x))\,dx$

Solution. $\int_7^5 (4g(x)-2f(x))\,dx$

$$= 4\int_7^5 g(x)dx - 2\int_7^5 f(x)\,dx$$

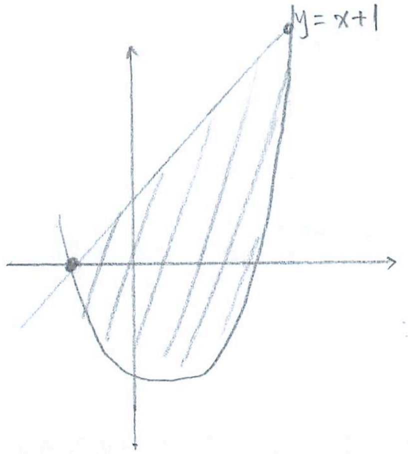
$$= 4\int_7^2 g(x)\,dx + 4\int_2^5 g(x)\,dx + 2\int_5^7 f(x)\,dx$$

$$= -4\int_2^7 g(x)\,dx + 4\int_2^5 g(x)\,dx + 2\int_5^7 f(x)\,dx$$

$$= -4\cdot\frac{5}{3} + 4\cdot 2 + 2\cdot 6$$

$$= \boxed{\frac{40}{3}}$$

13). Solution.



Find the two intersections, $x^2-4x-5 = x+1$

$$x^2-5x-6=0$$

$$(x-6)(x+1)=0$$

$$x=6 \qquad x=-1.$$

$$\text{Area} = \int_{-1}^{6} (x+1)-(x^2-4x-5)\,dx$$

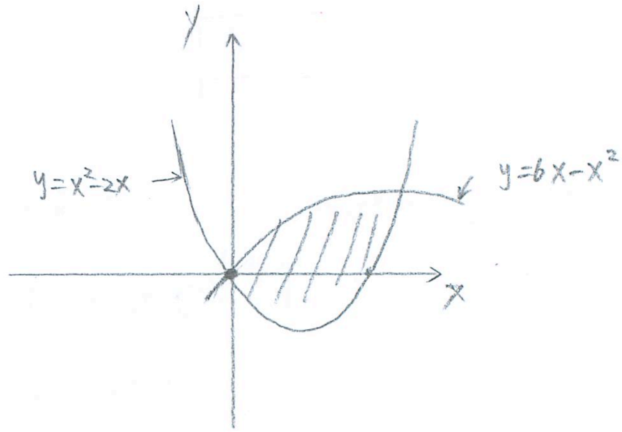
$$= \int_{-1}^{6} 6+5x-x^2\,dx$$

$$= 6x+\frac{5x^2}{2}-\frac{x^3}{3}\Big]_{-1}^{6}$$

$$= 54-\left(-\frac{19}{6}\right)$$

$$= \boxed{\frac{343}{6}}$$

14) Solution.



Find the two intersections,

$$x^2-2x=6x-x^2$$

$$2x^2-8x=0$$

$$2(x-4)x=0$$

$$x=0 \qquad x=4$$

$$\text{Area} = \int_{0}^{4} (6x-x^2)-(x^2-2x)\,dx$$

$$= \int_{0}^{4} 8x-2x^2\,dx$$

$$= 4x^2-\frac{2}{3}x^3\Big]_{0}^{4}$$

$$= \boxed{\frac{64}{3}}$$

15) Solution.

$$D'(p) = -\frac{2000}{p^2} \qquad D(5) = 832.$$

$$D(p) = \int -\frac{2000}{p^2}\,dp$$

$$= \int -2000\,p^{-2}\,dp$$

$$= 2000\,p^{-1} + C.$$

$$832 = 2000(5)^{-1} + C$$

$$C = 432$$

So $\boxed{D(p) = 2000\,p^{-1} + 432}$

16). Solution.

$$\frac{dN}{dt} = k(500 - N)\ ; \quad N(0) = 0\ ; \quad N(10) = 100.$$

Substitution, $y = 500 - N$ i.e. $N = 500 - y$.

$$\frac{dy}{dN} = -1.$$

Thus $\frac{dy}{dt} = \frac{dy}{dN}\frac{dN}{dt} = (-1)\,k\,(500 - N) = -ky.$

So, $y = Ce^{-kt}$

$$N = 500 - Ce^{-kt}$$

$N(0) = 0$, $\quad 0 = 500 - Ce^{-k\cdot 0} = 500 - C.$

$$C = 500.$$

$N(10) = 100$, $\quad 100 = 500 - 500\,e^{-k\cdot 10}$

$$e^{-10k} = \frac{4}{5}$$

$$-10k = \ln\frac{4}{5}$$

$$k = 0.022$$

Thus $\boxed{N = 500 - 500\,e^{-0.022t}}$

17). Solution.

$$\text{Total} = \int_0^7 s(t)\,dt$$

$$= \int_0^7 9 - 10e^{-0.3t}\,dt$$

$$= 9t - \frac{10}{-0.3} e^{-0.3t} \Big]_0^7$$

$$= \left(9\cdot 7 + \frac{10}{0.3} e^{-0.3\cdot 7}\right) - \left(9\cdot 0 + \frac{10}{0.3} e^{-0.3\cdot 0}\right)$$

$$= 33.749$$

$$\text{Average} = \frac{\text{Total}}{7}$$

$$= \boxed{4.821} \text{ (thousands).}$$

So 4821 cheeseburgers each day.