Finals, Math 804 Tanveer

Due date: Monday, Dec. 5th, 2011, 12 noon sharp

Instruction: No material outside class notes, homeworks and solutions. No collaboration/discussion. Any question to be directed to me alone.

1. Use contour integration to prove that

$$\frac{1}{\sin^2 z} = \sum_{n=-\infty}^{\infty} \frac{1}{(z - n\pi)^2}$$

2. Find expression for a conformal map $z(\zeta)$ from the exterior of the unit circle centered at the origin in the ζ plane to the interior of a square with unit sides in the $z-plane$ with center at the origin. Be specific on the location of preimages of the vertices of the square and conditions to determine unknown parameters.

3. Suppose $\Phi(z) = \frac{1}{2\pi i}\int_C \frac{\phi(s)}{s-z}ds$, with ϕ Holder continuous on C. Prove Plemelj formula, when C is an arbitrary smooth simple curve of finite length:

$$\Phi_{\pm}(t) = \frac{1}{2\pi i} \fint_C \frac{\phi(s)}{s-t}ds \pm \frac{1}{2}\phi(t)$$

4. Solve for $f(x)$ for $x \in (0,1)$ satisfying:

$$\int_0^x \frac{f(t)dt}{\sqrt{x-t}} + A\int_x^1 \frac{f(t)dt}{\sqrt{t-x}} = 1$$

where A is a real positive constant.