Homework 1, Math 804, Due Wednesday October 5th

1. Assuming $f(z)$ to be analytic, derive necessary relation between derivatives of $u(r,\theta)$ and $v(r,\theta)$ where $f(re^{i\theta}) = u(r,\theta) + iv(r,\theta)$.

2. Prove that satisfying Cauchy Riemann conditions along with continuity of first derivatives of $u(x,y)$ and $v(x,y)$ in a region R, implies that that $f(x+iy) \equiv u(x,y) + iv(x,y)$ is an analytic function of $z = x + iy$ in R. **Hint**: Express $f(z+h) - f(z)$ in terms of derivaties of u and v.

3. Prove that if $f(z)$ is continuous and $\oint_C f(z)dz = 0$ for <u>a</u>ny contour C in a region R, then $f(z)$ must be analytic. (**Hint:** You may want to consider $F(z) \equiv \int_{z_0}^{z} f(z)dz$). This is called Moerara's theorem.

4. Prove that a bounded analytic function that grows no faster than a polynomial function of order n as $z \to \infty$ must be an n-th order polynomial.

5. Determine function $\phi(x,y)$ that is harmonic and single valued at every point in $x^2 + y^2 \leq 1$, except at $(x_0, 0)$ where $\phi(x,y) = \ln\ \left[(x - x_0)^2 + y^2\right] + O(1)$ and satisfies $\phi = 0$ on $x^2 + y^2 = 1$. **Hint**: Note, if we define $\phi(x,y) = \ln\ \left[(x - x_0)^2 + y^2\right] + \psi(x,y)$, then $\psi(x,y)$ is single valued and harmonic).

6. Complete the proof of Phragmen-Lindelof principle stated in Week 2 notes.