Homework 1, Math 804 Tanveer
Due date: Monday, Oct. 5, 2009

1. Prove *Moerara's* theorem: If $f(z)$ is continuous in a simply connected region R and $\oint_C f(z)dz = 0$ for any closed path C contained entirely in R, then f is analytic in R.

2. Suppose $z = re^{i\theta}$ and $f(z) = u(r,\theta) + iv(r,\theta)$, with u, v real valued. Show that the Cauchy Riemann conditions in polar coordinates become:

$$u_r = \frac{1}{r} v_\theta \quad , \quad v_r = -\frac{1}{r} u_\theta,$$

   where subscript indicates partial derivatives.

3. Show that relation (19) on page 7 of week 1 notes is valid under suitable conditions on C.

4. Determine the maximum value of $\sin z$ on a circle of radius 2. (**Hint:** You might find it helpful to consider series representation)

5. Show that if f is analytic everywhere in $\mathbf{C}$, *i.e. entire*, with $|z^{-n} f(z)|$ bounded for some integer $n > 0$, then f can at most be a polynomial of degree n. (**Hint:** Note how Liouville's theorem was proved.

6. Complete the proof of Phragmen-Lindelof principle stated in Week 2 notes.