

Homework 4, Math 804 Tanveer
Due date: Wednesday, Nov. 9, '07

1. Use Schwartz Reflection principle to show that an analytic function f in a domain D can be analytically continued across an analytic segment of its boundary ∂D , where $\Im f$ (or $\operatorname{Re} f$) is a constant and the intersection of D with a sufficiently small ball centered at a boundary point is a "semi-circular" type domain on one side of ∂D . (Recall an analytic curve is the graph of some analytic function $\Gamma(t)$ for t real, $a < t < b$. You may also assume that $\Gamma'(t) \neq 0$, as appropriate for given assumption.)
2. Determine a mapping function that maps the upper-half plane into the interior of an equilateral triangle with sides of unit length. State any constraint(s) to determine parameters in the mapping.
3. Solve the potential problem for $\phi(x, y)$

$$\Delta\phi = 0 \text{ in } y > 0$$

with

$$\phi(x, 0) = 1 \text{ for } x > 1, \phi(x, 0) = -1 \text{ for } x < -1 \text{ and } \frac{\partial\phi}{\partial y}(x, 0) = 0 \text{ for } |x| < 1$$

State and prove conditions that will make the solution you find unique. **Hint:** Think of mapping to a semi-infinite strip and Schwarz Reflection Principle.

4. Complete the proof of Lemma 11.1 in Week 5 notes, page 1.
5. **a.** Find a Mobius transformation that maps the unit disk centered at 0 to the upper-half plane. **b.** Find a 1-1 analytic mapping of the interior of $|z - 1 - 2i| = 2$ to the interior of the unit disk centered at $i/2$.