**Homework 5, Math 804 Tanveer**
**Due date: Wednesday, November 18th, '09**

**1.** Consider the Fourier Transform of $f(x)$

$$\int_{-\infty}^{\infty} f(x)e^{ikx}dx$$

For $f(x) = \frac{1}{\sqrt{(x+1)^2+1}}$, determine the asymptotic series as $k \to \pm\infty$ and explicitly calculate the first two terms. **Hint**: Use Contour deformation in the complex $x$ plane and use Watson's Lemma appropriately.

**2.** Determine the asymptotic behavior of the Fourier Transform for large $|k|$ a more general real valued function $f(x)$ that is analytic on the real axis and has nearest singularities from the real axis in the form of branch points at $x = a \pm ib$, for $b > 0$, such that for some $\beta \in \mathbb{R}$,

$$f(x) \sim \frac{c}{(x-a-ib)^\beta} \text{ as } x \to a+ib, \text{ while}$$

$$f(x) \sim \frac{c^*}{(x-a+ib)^\beta} \text{ as } x \to a-ib$$

You may assume that except for singularities at $a \pm ib$, $f$ is analytic in $\{|\Im\, z| < \nu\}$ for some $\nu > b$.

**3.** Prove that the most general $\Phi(z)$ of degree $m$ at $\infty$, that satisfies

$$\Phi_+(t) = g(t)\Phi_-(t)$$

on a smooth closed curve $\mathcal{C}$ for a nonzero differentiable $g(t)$ is:

$$\Phi_+(z) = P_{m+k}(z)e^{\Psi_+(z)} \text{ for } z \text{ inside } \mathcal{C}$$

$$\Phi_-(z) = P_{m+k}(z)z^{-k}e^{\Psi_-(z)} \text{ for } z \text{ outside } \mathcal{C}$$

where $k = ind_{\mathcal{C}}g$, $P_{m+k}$ denotes a polynomial of degree $m+k$ and $\Psi_\pm$ is as defined in notes.

**4.** Determine solution to the following singular integral equation for $x \in (-1, 1)$:

$$f(x) + \fint_{-1}^{1} \frac{f(s)}{s-x}ds = x$$