

Homework 5, Math 805, Due: March 8th

1. Prove Proposition 3.210 (page 74 Text) for the oscillatory case $F^2 < 0$
2. Prove Proposition 3.210 (page 74 Text) for the case $F < 0$.
3. Consider the two point boundary value problem

$$\epsilon y'' + a(x)y' + b(x)y = 0 \quad , \quad y(0) = A, \quad y(1) = B$$

for $a(x), b(x)$ smooth with $a(x) > 0$ for $x \in [0, 1]$ and $0 < \epsilon \ll 1$. Show with appropriate transformation that WKB results produce the same results as formal boundary layer results.

4. Consider the two point boundary value problem

$$\epsilon^2 y'' - (1 - x^2)y = -1, \quad y(-1) = 0 = y(1)$$

for $x \in (-1, 1)$. Carry out a formal inner-outer expansion for small ϵ upto $O(\epsilon)$ and show in this case we have boundary layers only at end points $x = \pm 1$ and no where else. Carry out appropriate matching to determine all unknown constants needed to determine solution upto and including $O(\epsilon)$. (**Hint:** In this inhomogeneous problem, you may have to rescale dependent variable as well in the boundary layers.)