Finals, Math 805, Due: March 17th

1. Generalize Proposition 3.24 on page 29 in the text when when we have one interior maximum at $s = 0$ with $g'(0) = 0 = g''(0) = g'''(0)$, but $g^{(iv)}(0) < 0$. How does the argument change if the first nonzero derivative is $g^{(2n)}(0)$ (not asking for a full proof) ?

2. Determine leading term of the asymptotic series for

$$f(x) = \int_{\infty e^{-i\pi/5}}^{\infty e^{i\pi/5}} e^{-xt+t^5/5} dt$$

as $x \to \infty$. Following the cue from Airy's equation which has solution in similar form, determine a linear differential equation satisfied by f. Determine integral representations in the same form for other independent solutions to this ODE by changing limits.

3. Prove that there exists unique solution to the problem

$$y' - y = -\frac{1}{x^2} + y^4, \quad \text{with } y \to 0 \text{ , when } x \to +\infty$$

for sufficiently large x. Prove that this has an asymptotic series in powers of $\frac{1}{x}$ starting with $\frac{1}{x^2}$.

4. Prove that Proposition 3.210 in the text for WKB solution for

$$\epsilon^2 \psi'' - U(x, \epsilon)\psi = 0$$

holds when

$$U(x, \epsilon) = U_0(x) + \epsilon^2 U_1(x)$$

for smooth nonzero $U(x, \epsilon)$ (you may restrict your proof to $U > 0$.