April 8, 2013

Math 3345

Michael Tychonievich
Spring Semester 2013

Practice Midterm 2

Respond to the following five problems on your own paper. You should time yourself, giving yourself 55 *consecutive minutes* to complete the entire practice test. Do not use your notes. As this practice test is intended as a diagnostic tool, you will be assigned points based on whether or not you made a reasonable attempt at doing each problem, and not wholly on correctness. As such, we will go over this practice test in class only after it has been handed in on April 8.

1. **Prove or Disprove.** For any subset $A \subseteq \mathbb{N}$, if $A \neq \emptyset$ then A has a least element.

2. **Prove or Disprove.** Let A, B, C, and D be sets such that C and D are disjoint, and let $f : A \to C$ and $g : B \to D$ both be injective functions. Let $X = A \cup B$ and let $Y = C \cup D$. Define h by

$$h(x) := \begin{cases} f(x), & \text{if } x \in A \\ g(x), & \text{if } x \in B. \end{cases}$$

   Then $h : X \to Y$ is a function and h is injective.

3. **Prove or Disprove.** Let A and B be sets and let $f : A \to B$ be a function. If there is a function $g : B \to A$ such that $g \circ f = \mathrm{id}_A$, then g is the unique function with this property.

4. **Prove or Disprove.** Let S be a set and let $\mathscr{M}$ be a nonempty collection of subsets of S. Let $\mathscr{N} = \{S \setminus M : M \in \mathscr{M}\}$. Then $S \setminus \cup\mathscr{M} = \cap\mathscr{N}$.

5. **Prove or Disprove.** Let A and B be sets. Then the sets $A \times B$ and $B \times A$ are equinumerous.
   Note 1. For this problem, you may indicate that X is equinumerous to Y by writing $X \sim Y$ without further comment.
   Note 2. Be especially careful about how you handle the cases where either A or B is the empty set.